FORMATION OF NONCIRCULATING REGIONS IN THE AXIAL ZONE OF A VORTEX CELL

M. V. Bashkatov

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As is well known [1], in many cases twist flows are accompanied by the development of secondary flows (return flows for the case of twist jets or central zones in cyclone equipment). For a homogenous flow twist in accordance with the rigid-body law at the entrance the possibility of development of an axial reverse flow in the initial segment of the tubes was shown analytically in [2]. Some interesting results from the numerical computation of the flow in end-to-end tubes, one of which rotates and the other of which is at rest, are given in [3]. It is found that if the first (along the path of the flow) tube is at rest, a stagnant zone forms in the region near the wall, while if the second tube is at rest, this zone lies near the axis. Here and in the following by stagnant zones we mean regions of closed circulation flow into which the stream does not penetrate.

The development of secondary flows in the presence of twist is explained by the breakdown of the equilibrium between the pressure and centrifugal forces. For example, during the rotation of a disk in a fluid at rest [1] the fluid particles at the surface of the disk experience an increase in the azimuthal velocity due to viscous friction, and since the pressure penetrating from the volume of the fluid does not compensate for the centrifugal force, the particles are ejected in the radial direction. Different mechanisms are known for the formation of central cavities in twist flows [4].

In the present work we discuss certain characteristics of flow fluid in a swirling chamber (Fig. 1). It is shown that if the fluid is viscous, incompressible, and nondropping, then the formation of the stagnant zones in the axial regions of the swirling chamber occurs due to diffusion from the axis of symmetry. The swirling chamber can be divided into cells (in Fig. 1 the vortex cell is delineated by dashed lines) and the flow entirely investigated in the unit vortex cell, since the pattern is symmetrical with respect to the plane $z = \pm l(n+1/2)$.

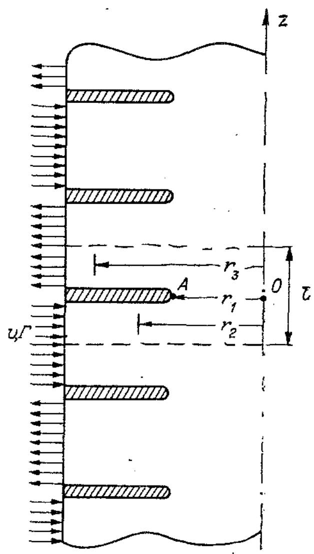

Fig. 1

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The investigations were carried out on the basis of a numerical solution of the complete Navier–Stokes equations in the new unknowns ξ, ψ, and Γ for the axisymmetric case. The equation for the azimuthal component of the curl of the velocity ξ is the transfer equation for the azimuthal component of the vorticity obtained by applying the operator curl_φ to the momentum-transfer equation. The equation for the circulation Γ is obtained directly from the equation for the azimuthal component of the velocity by a simple transition from the unknown v_φ to rv_φ. The stream function ψ is introduced in such a way that the equation of continuity is automatically satisfied. The equation for ψ is obtained from the definition of ξ, when the radial and axial components of the velocity written in terms of ψ are substituted into it. In the nondimensional form the initial system of equations has the following form:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial z}, \quad v_z = -\frac{1}{r}\frac{\partial\psi}{\partial r}, \quad \Gamma = rv_\varphi; \tag{1}$$

$$\frac{\partial\xi}{\partial\tau} + v_r\frac{\partial\xi}{\partial r} + v_z\frac{\partial\xi}{\partial z} - v_r\frac{\xi}{r} - \frac{2}{k^2}\frac{\Gamma}{r^3}\frac{\partial\Gamma}{\partial z} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2\xi}{\partial r^2} + \frac{1}{r}\frac{\partial\xi}{\partial r} - \frac{\xi}{r^2} + \frac{\partial^2\xi}{\partial z^2}\right); \tag{2}$$

$$\frac{\partial\Gamma}{\partial\tau} + v_r\frac{\partial\Gamma}{\partial r} + v_z\frac{\partial\Gamma}{\partial z} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2\Gamma}{\partial r^2} - \frac{1}{r}\frac{\partial\Gamma}{\partial r} + \frac{\partial^2\Gamma}{\partial z^2}\right); \tag{3}$$

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = r\xi. \tag{4}$$

The scales of the quantities are defined as follows: r_1 is the radius of the critical point A; v_{r_1} is the velocity in the section $r=r_1$ in the presence of a uniform radial flow; and Γ_0 is the circulation at the entrance. The flow in the chamber is determined by the following parameters:

$$L = l/r_1, \quad \mathrm{Re} = v_{r_1}r_1/\nu, \quad k = v_{r_1}r_1/\Gamma_0,$$

where Re is the Reynolds number and k is the twist parameter.

The symmetry of the flow pattern with respect to the surfaces $z=L/2$ and $z=-L/2$, which are not solid surfaces, permits formulation of the boundary conditions of zero tangential stresses and impenetrability: $\partial\Gamma/\partial z = 0$, $\xi = 0$, $\psi = 0$.

Analogous boundary conditions have to be satisfied at the intermediate projection: $\partial\Gamma/\partial z = 0$, $\xi = 0$, $\psi = -c$. The formulation of such artificial boundary conditions is justified due to the following: First, the flow in the axial region is of primary interest in the problem; second, it is assumed that for a given flow pattern the effects of viscous interaction of the flow with the solid wall and of flow separation will be localized and concentrated near the intermediate projection, having little influence on the flow pattern in the axial region; third, the neglect of viscous interaction of the flow with the solid wall, i.e., the replacement of no-slip boundary conditions by the conditions of zero tangential stresses, appreciably simplifies the numerical computation of the flow in the chamber. For the same reasons we require that $\xi=0$ at the critical point. The conditions of symmetry and quasirigid rotation are specified at the axis: $\Gamma=0$, $\xi=0$, $\psi=0$. The conditions of uniformity of the flow velocity profile are set up at the entrance to the vortex cell: $\psi=-c(L+2z)/L$, $\xi=0$, $\Gamma=1$. It is assumed that in the exit section, sufficiently remote from the central region of the chamber, v_r and Γ profiles are equalized and $v_z \to 0$; the circulation at the exit is taken to be the same as at the chamber entrance: $\psi = +c(2z - L)/L$, $\xi = 0$, $\Gamma = 1$.

We not get back to the initial system of equations. It is evident from Eq. (2) that if the circulation changes in the axial direction, then sources of an azimuthal vorticity component appear in the volume of the fluid, i.e., we get the term $\Gamma/r^3 \cdot \partial\Gamma/\partial z$. In this case the equilibrium between the pressure and centrifugal forces is also disturbed. The change in the field ξ in turn leads to a change in the field ψ or correspondingly in v_r and v_z. The effect of twisting is characterized by the number k. The quantity $1/k^2$ is the ratio of the characteristic scales of the inertial and centrifugal forces. In the case of an ideal fluid the axial gradient of the circulation appears in the presence of radial displacements, if the circulation profile is inhomogeneous in the entrance section [5]. In the case of viscous fluid large values of $\partial\Gamma/\partial z$ may appear due to the interaction of the flow with the solid wall. In the present problem the boundary conditions formulated above exclude both these cases. In certain cases the diffusion of the axial component of the vorticity from the axis of symmetry may play a role in the development of $\partial\Gamma/\partial z$.

A simple example will show the possibility of diffusion of the z component of the vorticity from the axis of symmetry. Let the profiles of the radial and axial velocity components in a cylindrical channel have the form $v_r=-r$, $v_z=2z$. If the flow is twisted and $\Gamma=1$ at the surface $(r=1)$, then the solution of Eq. (3) satisfying the condition of quasirigid rotation at the axis will be

$$\Gamma = \left[1 - \exp\left(-\frac{\mathrm{Re}\, r^2}{2}\right)\right] \Big/ \left[1 - \exp\left(-\frac{\mathrm{Re}}{2}\right)\right]$$

and the axial component of the vorticity $\omega_z = 1\partial\Gamma/r\partial r$ has the form

$$\omega_z = \mathrm{Re}\, \exp\left(-\frac{\mathrm{Re}\, r^2}{2}\right) \Big/ \left[1 - \exp\left(-\frac{\mathrm{Re}}{2}\right)\right].$$

This solution shows that under the action of viscous stresses an element of the fluid approaching the axis will lose its torque, imparting it to the outer layer. The effect of the viscous mechanism of radial torque transfer (or diffusion of the vorticity from the axis of symmetry) is decisive in the Rank effect, i.e., the vortex effect of temperature separation of a gas [6]. Here an energy transfer from the inner layers of the fluid to the outer layers occurs simultaneously with the viscous mechanism of torque transfer, since the inner layers untwist the outer.

In a vortex cell (Fig. 1) convective transfer also occurs along with the viscous mechanism of torque transfer. The simultaneous action of viscous and convective torque transfer results in a breakdown of the homogeneity of the circulation profile along z. This can be illustrated for the case of small twists $k \gg 1$. In the zero-order approximation $\xi = 0$ and in the central region $(0 \le r < 1, -L/2 \le z \le L/2)$ we have

$$\psi = \sum_{n=1}^{\infty} c_n r I_1 \left[\frac{n\pi r}{L}\right] \cos\frac{n\pi z}{L}, \qquad (5)$$

where I_1 is a modified Bessel function. In the axial region only the first term of series (5) need be retained and, based on the behavior of the function I_1 for $r \to 0$, the flow can be expressed in the form

$$v_r = \frac{\pi c_1}{L} r \sin\frac{\pi z}{L}, \quad v_z = 2c_1 \cos\frac{\pi z}{L}. \qquad (6)$$

In the boundary-layer approximation, when viscous exchange in the axial direction is disregarded, the solution of Eq. (3) for the given velocity profile (6) has the form

$$\Gamma = 1 - \exp\left[-\frac{\mathrm{Re}' r^2}{2} \frac{\sqrt{1-y^2}}{2(\pi - \arccos y)}\right], \qquad (7)$$

where $y = \sin(\pi z/L)$; $\mathrm{Re}' = \mathrm{Re}\, c_1 \pi/L$.

The distribution of the circulation in the axial region as given by (7) is shown in Fig. 2. The solid lines denote the constant value of circulation for $\mathrm{Re}' = 1000$; the dashed lines denote the same for $\mathrm{Re}' = 10{,}000$ (curves 1-7 correspond to the values $\Gamma = 0.99$, 0.9, 0.7, 0.5, 0.3, 0.1, 0, respectively). The nature of the circulation distribution in the axial region remains practically unchanged if the viscous torque transfer in the axial direction is taken into consideration. In this case the constant-circulation curves will not go to infinity $(r \to \infty)$ for $z \to L/2$ and will close at the surface $z = L/2$.

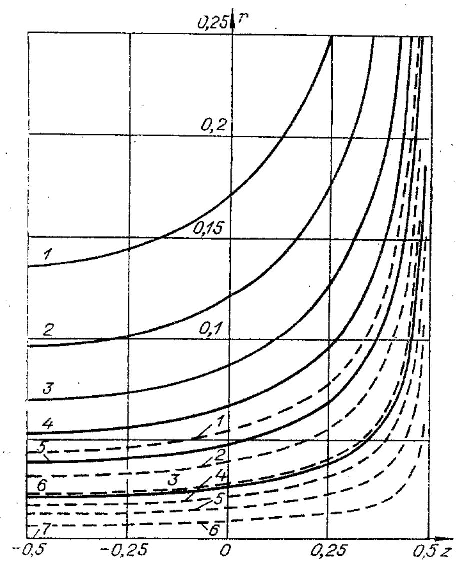

Fig. 2

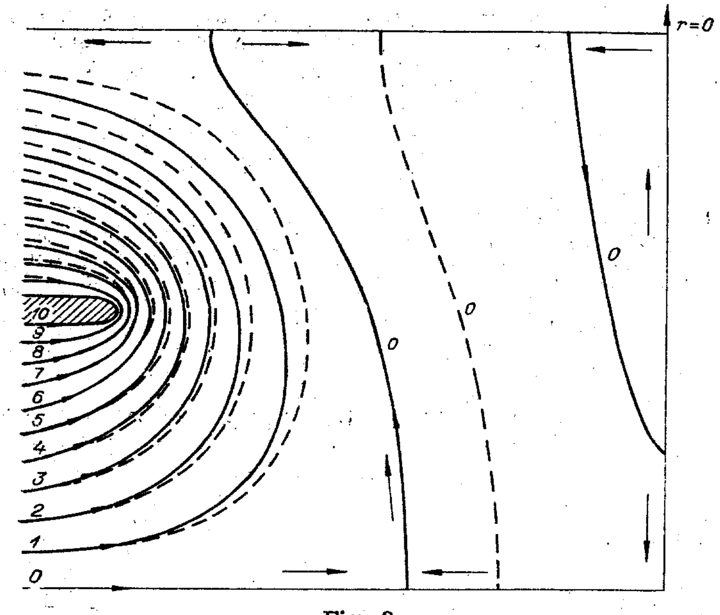

Fig. 3

If we now turn to the transfer equation for ξ, then it is obvious that a negative gradient of the circulation in the axial direction will lead to the development of vorticity $\xi < 0$. Due to the viscous and convective transfer the induced vorticity ξ extends over the region of the flow and accumulates at the boundary. The negative values of the azimuthal vorticity component correspond to the clockwise twisting of the flow in the r, z plane (see Fig. 1) and to a backing-off of the flow to the periphery. Therefore, the formation of a stagnant zone near the axis may be expected for sufficiently large values of Re and $1/k^2$.

The numerical computations of the twist flow in a vortex cell carried out here show stagnant zones near the axis and the validity of their formation mechanism described above. The distributions of the stream function obtained from the numerical solution are shown in Fig. 3. The dashed lines denote the lines of constant values of the stream function for Re = 25; the continuous curves denote the same for Re = 100 (curves 0–10 correspond to the values of ψ from 0 to −0.5 at 0.05 intervals). The computations were carried out for the following values of the parameters: k = 1, L = 1, $r_2/r_1 = 2$, $r_3/r_1 = 4$ (r_2 and r_3 are the radii of the surfaces on which the conditions, set up at the entrance and exit from the chamber, are applied).

As seen from Fig. 3, the zone thus formed expands with the increase in Re. The intensity of the induced circulation in it also increases with the growth of the stagnant zone. It is obvious that from a certain instant the action of the mechanism described above for the formation of the first zone leads to the appearance of a second stagnant zone. The diagram for the distribution of ψ for Re = 100 illustrates this case. With further increase of Re the second zone will expand and a third zone may form. At very large Re the flow probably becomes turbulent in the central region and these zones are reduced to a single zone.

In order to determine the stationary flow in the vortex cell the system of equations (1)–(4) with the corresponding boundary conditions was solved numerically by the relation method, for which the term $r\partial\psi/\partial r$ is added to Eq. (4). An implicit difference scheme

$$\frac{f_i^{n+1/2} - f_i^n}{\tau} + L_i\left(A_r f_i^n,\ A_z f_i^{n+1/2},\ A_r^2 f_i^n,\ A_z^2 f_i^{n+1/2}\right) = 0,$$

$$\frac{f_i^{n+1} - f_i^{n+1/2}}{\tau} + L_i\left(A_r f_i^{n+1},\ A_z f_i^{n+1/2},\ A_r^2 f_i^{n+1},\ A_z^2 f_i^{n+1/2}\right) = 0.$$

was constructed by the method of fractional steps similar to [7]; here f_i denotes Γ, ξ, and ψ, respectively; $A_r f_i$, $A_z f_i$, $A_r^2 f_i$, and $A_z^2 f_i$ are, respectively, the first and second central difference derivatives of the function f_i with respect to r and z. It should also be noted that in the difference equations for f_i all other unknown functions f_j, where $i \neq j$, are taken from the preceding layer. This permits the system of equations to be decoupled and the distributions of Γ, ξ, ψ in the (n+1)-th layer to be determined successively. Additional integrations were carried out in the solution of the difference equation for ψ. The Dirichlet boundary conditions were approximated explicitly, while the Neumann boundary conditions were approximated according to [7] starting from the expression

$$\Gamma[i,1]=\Gamma[i,0]+\frac{\partial\Gamma}{\partial z}\bigg|_{i,0}h_z+\frac{\partial^2\Gamma}{\partial z^2}\bigg|_{i,0}\frac{h_z^2}{2}.$$

The value of the second derivative is found directly from the equation for Γ extended to the boundary. A 40 × 40 grid uniform over z and nonuniform over r was used for the solution in the central region. The r-variation of the computing step was specified in the following way:

$$r[i]=h_r[0]\frac{(1+\alpha)^i-1}{\alpha}\quad\text{or}\quad h_r[i+1]=(1+\alpha)h_r[i].$$

For the computations discussed above $\alpha=0.024$.

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SECONDARY FLOWS BESIDE A CYLINDER IN A COMPLEX SOUND FIELD

V. B. Repin

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It is known that steady flows arise beside a solid surface in the presence of a sound field which can to a certain extent exert an effect on the processes of heat and mass exchange [1-3]. As a rule, all papers from this area refer to the case in which one can represent the sound field in the form of a single wave. However, situations are often encountered in practice in which the sound field is complex; i.e., it consists of several vibrations whose amplitudes and frequencies are unlike in the general case. The secondary flows which form beside a circular cylinder placed in a complex soundfield are investigated in this paper.

Let n plane waves with the following parameters encounter a circular cylinder of radius R: A_n is the velocity amplitude of the acoustic shift in the n-th wave, ω_n is the frequency, a_n is the point of encounter of the wave with the cylinder, and φ_n is the phase of the wave. Let us consider the case in which the radius of the cylinder is significantly less than the wavelength; then the flow beside the cylinder can be treated as incompressible.

The Navier–Stokes equation describing the motion of a viscous incompressible liquid has the form

$$\frac{\partial}{\partial t}(\nabla^2\psi)-\varepsilon\frac{\partial(\psi,\nabla^2\psi)}{\partial(\theta,r)}=\frac{1}{2}\mathrm{H}^2\nabla^4\psi,\tag{1}$$

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